OCR
Oxford Cambridge and RSA

Monday 12 May 2025 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

18652388663

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}.$$

(a) Use **A** and **B** to show that matrix multiplication is **not**, in general, commutative. **[2]**

(b) Verify that **A** and **B** satisfy $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. **[3]**

2 **In this question you must show detailed reasoning.**

Find the acute angle between the vector $3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and the normal vector to the plane $2x+3y+z=6$. **[4]**

3 The matrices **M** and **N** are given by

$$\mathbf{M} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix}$$ where a and b are positive constants.

(a) Given that $\mathbf{M}^2 = \mathbf{N}$, determine the exact values of a and b. **[4]**

(b) Hence state the transformations of the plane associated with matrices **M** and **N**. **[3]**

4 **(a)** The transformation T is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix}$.

A shape S_1 is mapped to a shape S_2 by the transformation T.

Show that volume of S_1 is the same as the volume of S_2. **[2]**

(b) Three planes have equations

$$\begin{aligned} x - 2y + 2z &= \lambda, \\ 2x + y &= 2, \\ x + 2y - z &= 0, \end{aligned}$$

where λ is a constant.

(i) Explain why the three planes intersect at a point for any value of λ. **[2]**

(ii) Use a matrix method to determine, in terms of λ, the coordinates of this point. **[4]**

5 **In this question you must show detailed reasoning.**

The complex number w is given by $w = -4\sqrt{2} + \left(4\sqrt{2}\right)\mathrm{i}$.

(a) **(i)** Find $|w|$. **[2]**

(ii) Find $\arg(w)$. **[2]**

The complex numbers z_1 and z_2 are given by $z_1 = a + \mathrm{i}$ and $z_2 = 4(\cos\theta + \mathrm{i}\sin\theta)$, where a is a positive real constant and $-\pi < \theta \leqslant \pi$.

(b) You are given that $z_1 z_2 = w$.

(i) Find the exact value of a. **[3]**

(ii) Find the value of θ. Give your answer as an exact multiple of π. **[3]**

6 **(a)** Express $\dfrac{1}{(r-1)^2}-\dfrac{1}{(r+1)^2}$ as a single simplified fraction. **[2]**

(b) Hence determine the limit which $\displaystyle\sum_{r=2}^{n}\frac{r}{(r-1)^2(r+1)^2}$ converges to as $n\to\infty$. **[5]**

7 The region R of the Argand diagram consists of the set of points representing complex numbers z which satisfy the following inequalities.

$\mathrm{Im}(z)\geqslant 0$ $\qquad\arg(z+2)\leqslant\frac{1}{4}\pi$ $\qquad|z|\leqslant|z-4-2\mathrm{i}|$

(a) Sketch and clearly label the region R on an Argand diagram. **[4]**

(b) **In this question you must show detailed reasoning.**

Find the largest value of $\mathrm{Im}(z)$ in the region R. **[7]**

8 The three distinct roots of the equation $z^3-4z^2+pz+q=0$, where p and q are real, are drawn on an Argand diagram. The three points which represent these roots do not lie on a straight line but instead form a triangle T.

(a) Show that T is isosceles. **[3]**

(b) **In this question you must show detailed reasoning.**

You are given the following information.

- The area of T is 10 square units.
- One of the roots of the equation $z^3-4z^2+pz+q=0$ is $z=-2$.

Find the other roots of the equation. **[5]**

END OF QUESTION PAPER